8. Three Dimensional Geometry

- **Direction cosines (d.c.'s) of a line:**
  - d.c.'s of a line are the cosines of angles made by the line with the positive direction of the coordinate axes.
  - If l, m, and n are the d.c.'s of a line, then $l^2 + m^2 + n^2 = 1$
  - d.c.'s of a line joining two points P (x_1, y_1, z_1) and Q (x_2, y_2, z_2) are $\frac{x_2 - x_1}{PQ}, \frac{y_2 - y_1}{PQ}, \frac{z_2 - z_1}{PQ}$, where

    $$PQ = \sqrt{\left(x_2 - x_1\right)^2 + \left(y_2 - y_1\right)^2 + (z_2 - z_1)^2}$$

- **Direction ratios (d.r.'s) of a line:**
  - d.r.'s of a line are the numbers which are proportional to the d.c.'s of the line.
  - d.r.'s of a line joining two points P (x_1, y_1, z_1) and Q (x_2, y_2, z_2) are given by $x_1 - x_2, y_1 - y_2, z_1 - z_2$ or $x_2 - x_1, y_2 - y_1, z_2 - z_1$.

- If a, b, and c are the d.r.'s of a line and l, m, and n are its d.c.'s, then $\frac{l}{a} = \frac{m}{b} = \frac{n}{c}$

  $$l = \pm \frac{a}{\sqrt{a^2 + b^2 + c^2}}, m = \pm \frac{b}{\sqrt{a^2 + b^2 + c^2}}, \text{ and } n = \pm \frac{c}{\sqrt{a^2 + b^2 + c^2}}$$